Exercise 1: The neo-Hookean model predicts that the internal pressure p_i in a balloon is given by,

$$p_i(\lambda) = 4C_{10} \frac{e_i}{R} \frac{1}{\lambda} \left(1 - \frac{1}{\lambda^6}\right)$$

Here C_{10} is a constant, $\lambda = r/R$ is the stretch ratio and e_i, R are the initial thickness and radius of the balloon. Find the stretch ratio corresponding to maximum pressure in the balloon.

Exercise 2: A general constitutive relation in non-linear elasticity (for incompressible materials) proposed by Ogden, is,

$$\phi(\lambda_1, \lambda_2, \lambda_3) = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) \qquad \text{(a)}$$

where α_i, μ_i are parameters of the model and N is the number of terms considered in the function. It is chosen according to the experimental data. Show that this model,

(1) reduces to Neo-Hookean model when, $N = 1, \ \alpha_1 = 2, \ \mu_1/2 = C_{10}$ and
(2) reduces Mooney-Rivlin model when
$N = 2, \ \alpha_1 = 2, \ \alpha_2 = -2, \ \mu_1/2 = C_{10}, \ -\mu_2/2 = C_{01}$.